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Adding flavor to AdS/CFT

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Abstract: Coupling fundamental quarks to QCD in the dual string representation corresponds to adding the open string sector. Flavors therefore should be represented by space-time filling D-branes in the dual 5d closed string background. This requires several interesting properties of D-branes in AdS. D-branes have to be able to end in thin air in order to account for massive quarks, which only live in the UV region. They must come in distinct sets, representing the chiral global symmetry, with a bifundamental field playing the role of the chiral condensate. We show that these expectations are born out in several supersymmetric examples. To analyze most of these properties it is not necessary to go beyond the probe limit in which one neglects the backreaction of the flavor D-branes.

Keywords: 1/N Expansion, Confinement, AdS-CFT and dS-CFT Correspondence, QCD.

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Contents

1. Introduction

In the large-N limit the Feynman diagrams of $\mathrm{SU}(N)$ Yang-Mills theory reorganize themselves into a genus expansion of closed string theory [1]. This closed string is believed to propagate in a five dimensional background [2]. At each point on the worldvolume of the string, we have to specify its position in 4d Minkowski space and its thickness, which is represented by its position in the 5th dimension. The metric structure of the 5th dimension describes the internal structure of the string. This expectation has been realized in many supersymmetric examples starting with [3], where in addition to the 5th dimension, the closed string background includes an internal compact space, representing the additional fields in the theory. Thus, pure Yang-Mills, with only glue as degrees of freedom, is expected to map to an entirely 5d non-critical string theory background [2].

Adding fundamental flavors effectively introduces boundaries in the 't Hooft expansion, that is one adds an open string sector. Since the open strings should be allowed to have a thickness as well, one is led to believe that adding fundamental flavors in the gauge theory maps to adding space-time filling D-branes in the 5d bulk theory. In the limit where M, the number of flavors, is much smaller than N, the backreaction of the D-branes on the bulk geometry can be ignored. As we discuss later, this corresponds to the quenched approximation of lattice gauge theory. Several puzzles arise. The spacetime filling D-branes can't carry any charge in order to avoid tadpoles, but nevertheless should be stable. Making the quarks very heavy should decouple them from the IR theory, so D-branes dual to massive quarks should be spacetime filling in the UV region but than end at a finite distance in the 5th dimension, and be absent in the IR. The gauge fields living on the D-branes map to global flavor currents in the gauge theory. Since QCD with M flavors has a chiral $\mathrm{SU}(M) \times \mathrm{SU}(M)$ symmetry, which then gets broken by a condensate, one actually needs two sets of space-time filling branes each carrying one of the $\mathrm{SU}(M)$ symmetries. The chiral condensate should then manifest itself in the bulk as a vev for a scalar, which is bifundamental under the product gauge symmetry. We will show that all this is indeed the case in the supersymmetric cousins of QCD, where the dual closed string background is known.

In the next section we will consider the probe limit, and show how to introduce space-time filling branes without running into contradictions with tadpoles. Most of the stable D-branes we know carry charge under RR-gauge fields, so that without introducing orientifolds it is usually impossible to introduce space-time filling branes, since the net charge has to cancel. We avoid this problem by having our D-branes wrap topologically trivial cycles with zero flux, so that they carry no charge in the 5d world. They are stabilized by the same mechanism as in [4, 5]: there are negative mass modes which control the slipping of the D-branes off the cycle they wrap, but the mass is not negative enough to lead to an instability in the curved 5d geometry, e.g. for our conformal examples it is above the BF bound [6, 7]. We show how to read off both the cycle wrapped in the internal manifold, and the worldvolume scalars that are turned on, from knowledge of the embedding in the flat 10d spacetime. Giving a mass to the flavors turns on a non-trivial profile for the slipping mode, making the D-brane seemingly terminate from the 5d point of view. Last but not least, we will study in this section a simple model that has an $\mathrm{SU}(M) \times \mathrm{SU}(M)$ global symmetry and show how this is reproduced in the bulk by having two distinct spacetime filling sets of branes, which differ by the cycle they wrap in the internal space. We explicitly break the product global symmetry to the diagonal by turning on a mass for the quarks. In the corresponding bulk physics this amounts to Higgsing the gauge symmetry by a bifundamental scalar, mimicking the bulk physics dual to a chiral condensate. We will also verify that all our solutions are supersymmetric by analyzing κ-symmetry on the worldvolume.

Section 3 will be devoted to our most interesting example, a close cousin of $\mathcal{N} = 1$ SYM with M flavors. This theory actually has a chiral global symmetry. Again we will show how to obtain the $\mathrm{SU}(M) \times \mathrm{SU}(M)$ symmetry in the bulk from two sets of D-branes, but this time a single D7 brane will only give rise to a single chiral multiplet. As opposed to the examples studied before, the $\mathcal{N} = 1$ theory has non-trivial B field turned on in the supergravity background. So this time a lower dimensional D-brane charge is induced and tadpole cancellation becomes a non-trivial constraint, corresponding to the dual gauge theory being anomaly free. In section 4 we conclude and speculate about the use of D-brane probes to extract the non-perturbative superpotential of SQCD.

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2. Spacetime filling branes and tadpoles

2.1 The probe limit

In this work we only study branes in the probe limit, as in [4]. This is justified by taking the limit where $g_s \to 0$, M fixed, such that $g_s M \to 0$, which is the strength with which the M D-branes source the metric, dilaton and gauge fields. At the same time we take $N \to \infty$ holding as usual $g_s N = \lambda$ fixed. So the D3 brane backreaction is large, and we replace the D3 branes with their near-horizon geometry. The flavor probe branes will minimize their worldvolume action in this background without deforming the background. In particular this means that we take $M \ll N$, that is in the large-N gauge theory we introduce a finite number of flavors. In the lattice literature this limit is known as the quenched approximation: the full dynamics of the glue and its effect on the fermions is included, but the backreaction of the fermions on the glue is dropped. In the probe limit this approximation becomes exact. For some of the backgrounds we study, like the D3-D7 system, the supergravity solution is known beyond the probe limit [8, 9, 10, 11].

2.2 Tadpoles

As explained in the introduction, our proposal that fundamental quarks are dual to spacetime filling D-branes seems to suffer from a problem with tadpoles: usually stable D-branes carry charge, and for a spacetime filling brane, the net charge has to cancel. The way the supersymmetric branes we study avoid this pitfall is that they wrap topologically trivial cycles in the internal space. This phenomenon was encountered in [4] for the closely related D3-D5 system. There the field theory interpretation is in terms of a CFT with defect or boundary (dCFT). The M D5 branes give rise to M flavors in the dual gauge theory, but they are confined to a codimension one defect. In the supergravity dual, the branes aren't spacetime filling but live on an AdS_4 inside the AdS_5. For the cosmological constant to not jump across the branes, they also should not carry any D3 brane charge. This happens since the D5 wraps an equatorial, topologically trivial S^2 inside the S^5. Also the S^2 is not stabilized by any flux, so there is no induced D3 brane charge. The brane is stabilized by the dynamics of fields in AdS. As shown by Breitenlohner and Freedman [6], a negative mass mode in AdS_{d+1} does not lead to an instability as long as the mass is above the BF bound, $m^2 \geq -d^2/4$ in units of the curvature radius. Roughly speaking, this is due to AdS being a box. To satisfy the boundary conditions, all non-vanishing fluctuations have to carry a finite amount of gradient energy. In order to lower the energy of the system by rolling down the hill, the field's loss in potential energy would have to outweigh its gain in gradient energy. As shown in [4] the slipping mode that wants to have the D5 move off the equator and contract to a point, as allowed by topology, is such a stable, negative mass mode above the BF bound.

The same mechanism can be employed to get flavors from spacetime filling D-branes. The simplest example is the D3-D7 system. The corresponding gauge theory is $\mathcal{N} = 4$ SU(N) SYM with M additional fundamental $\mathcal{N} = 2$ hypermultiplets. This theory is asymptotically non-free, but the β-function is proportional to $g_{YM}^2 M$, so that in the probe limit the theory is still conformal, the background is just the usual $AdS_5 \times S^5$. Writing the S^5 as

$$ds^2 = L^2 \left(d\psi^2 + \cos^2(\psi) d\theta^2 + \sin^2(\psi) d\Omega_3^2 \right) \tag{2.1}$$

the D7 branes wrap an S^3 given by

$$\psi = 0 \tag{2.2}$$

which can easily be seen from its flat space embedding as we will explain below. Expanding the D7 brane action to quadratic order in the fluctuations of Z_ψ around the extremum $\psi = \pi/2$ (as was done in [12] for the D3 D5 system)

$$\mathcal{L} \sim 1 + \frac{1}{2}(\partial Z_\psi)^2 - \frac{3}{2} Z_\psi^2 \,, \tag{2.3}$$

we can read of the masses of scalar fluctuations and the corresponding operator dimensions

$$m_l^2 = l(l+2) - 3 \,, \qquad \Delta_l = 1 - l, 3 + l \,. \tag{2.4}$$

The dimension 3, $l = 0$, mode maps to the fermion mass term in the CFT. This spectrum was already anticipated in [8] without actually doing the calculation. It was noted that due to superconformal invariance, the r-charge and the dimension of the chiral operators are related, so that one can simply determine the dimension of a given spherical harmonic on the S^3 from its angular quantum numbers.

In order to establish that this S^3 is actually a supersymmetric solution to the worldvolume equations of motion, we have to check κ symmetry on the worldvolume. We will do so for this and similar configurations at the end of this section.

2.3 Worldvolume scalars: D-branes vanishing in thin air

In order to determine the above cycle on which the 7-branes wrap the internal manifold, we used our knowledge of the brane setup in the full asymptotic space-time. The coordinate

$$u^2 = x_4^2 + x_5^2 + x_6^2 + x_7^2 + x_8^2 + x_9^2 \tag{2.5}$$

becomes the radial coordinate in AdS$_5$ and the S^5 is the u = const. slice of the R^6 parametrized by the 4,5,6,7,8,9 coordinates. The D7 sitting on

$$x_8 = x_9 = 0 \tag{2.6}$$

wraps an equatorial S^3 inside the S^5 and fills all of AdS$_5$. Generically the cycle wrapped by the D7 brane will not have this simple product form, but the cycle in the internal space will be fibered over the AdS space. From the 5-dimensional point of view this is encoded in a non-trivial profile for one of the D-brane worldvolume scalars. To see how this works let us study the D3-D5 system of [4] with a non-zero mass term for the defect hyper. In [4] similar logic was used to establish that a D5 brane along $x_3 = x_7 = x_8 = x_9 = 0$, wraps the equatorial S^2 inside the S^5. In the AdS$_5$, whose metric is given by

$$ds^2 = L^2 \left(\frac{du^2}{u^2} + u^2 \left(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) \right) , \tag{2.7}$$

it lives on $x_3 = 0$; that is on an AdS$_4$ slice with the same curvature radius, stretching straight from the boundary towards the horizon. The field theory described by this brane system is 4d $\mathcal{N} = 4$ SYM with a 3d hypermultiplet defect [13, 4, 12].

The simple product form of the worldvolume metric gets modified when turning on a mass term for the defect hyper, by having the D5-brane now at $x_7 = c$. Using a parametrization of the S^5 where $x_7 = u\cos(\psi)$ this leads to

$$\psi = \arccos\left(\frac{c}{u}\right) \sim \frac{\pi}{2} - \frac{c}{u} - \frac{1}{6}\frac{c^3}{u^3} + \cdots, \tag{2.8}$$

where on the D5 worldvolume u only takes values between ∞ and c. For $c = 0$ we recover the equatorial S^2 sitting at $\psi = \pi/2$. Since ψ is one of the worldvolume scalars, we see that non-zero c induces a u-dependent profile for this scalar. The tension of the D5, that is the potential for ψ, is proportional to the radius of the S^2, $\cos^2(\psi)$. At $u = c$ the tension of the D-brane becomes zero and the D5 brane just stops. This scalar profile can be interpreted as an exactly solvable boundary RG flow.[1]

This at first sounds implausible, D-branes should not be able to end in the middle of nowhere. In our case, this is possible due to the existence of the BF bound in AdS: for one we have a stable D-brane without charge, so that the usual reasoning that the D-brane has to end on something which carries away its flux, fails. The fact that the tension can go to zero at a point in space relies on the fact, that we can have a scalar which lowers the tension without leading to an instability. From the higher dimensional point of view it is clear that the D-brane isn't really ending at all since the S^2 goes to zero size. Similar phenomena have appeared in the literature, e.g. in [15] where an internal circle shrinks when a D6 brane "ends". What is different there is that spacetime itself is a circle fibration. Compactifying on the internal space, the locus of the degenerate fiber becomes a defect, on which the D-brane then ends. In our case the background is a product and the S^5 has constant size. It is only the S^2 the brane wraps that vanishes. In the lower dimensional description the brane literally vanishes in thin air.

From the dual dCFT point of view, it is also clear what is going on. The leading c/u term means we deformed by the dimension 2 operator dual to ψ, so that the boundary behavior is like $\frac{1}{u}^{d-\Delta} = 1/u$. This is just the mass term for the defect hyper. The absence of the subleading $1/u^2$ term indicates that no vev is turned on. In the dCFT the defect has to become transparent at energies below the mass of the defect hyper. It is amazing to see how directly the gravity captures this phenomenon: the D-brane stops at the energy scale set by the mass of the defect hyper.

2.4 Interactions and the subleading term in AdS/CFT

The behavior of the D3-D5 defect system generalizes to the case of large-N gauge theory with massive flavors, e.g. the D3-D7 system, with the D7 brane separated from the D3 branes in x_8. The D-brane representing the massive fundamentals is spacetime filling in

[1] Recently a different boundary RG flow appeared in [14]. Their flow breaks all supersymmetry and corresponds to adding an operator of dimension 4. There, the actual embedding of the brane changes, and only asymptotically does it become AdS$_4$ $\times$ S^2.

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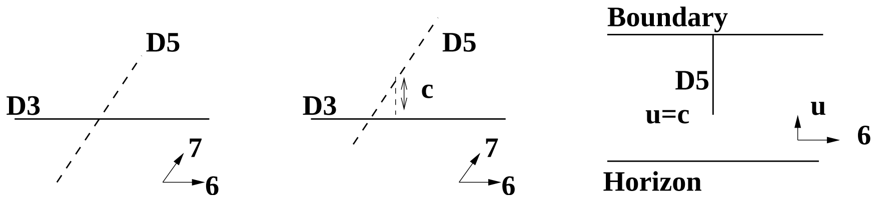

Figure 1: D3 D5 system with mass term for defect scalars, in the corresponding 5d AdS spacetime, the D5 brane ends in the middle of nowhere.

the UV and then simply stops at the position dual to the mass scale. The exact profile for ψ is once more known and given by (2.8). The leading term again corresponds to the fermion mass term, this time of dimension 3. However, here, the subleading order u^{-3} term seems to indicate the presence of a vev. We know from the field theory that this is not the case. We will show that this is an artifact of treating the scalar as a free field. According to the rules of AdS/CFT [16, 17], we have to evaluate the classical action on the solution. In the presence of a leading c/u term higher order terms from the non-linearities in the action, quartic in the leading term, come in at the same order as the first contribution of the subleading c^3/u^3 term. We use a coordinate $r = \log(u)$ in terms of which the metric reads

$$ds^2 = e^{2r}\left(-dt^2 + d\vec{x}^2\right) + dr^2 \tag{2.9}$$

In terms of the fluctuating field, defined via $\psi = \frac{\pi}{2} + \varphi(r)$, the DBI action expanded to quartic order in φ reads

$$S = \int d^5x e^{4r} \sin^3(\psi)\sqrt{1+(\varphi')^2} = \left(1 - \frac{3}{2}\varphi^2 + \frac{7}{8}\varphi^4 + \cdots\right)\left(1 + \frac{1}{2}(\varphi')^2 - \frac{1}{8}(\varphi')^4 + \cdots\right). \tag{2.10}$$

Evaluated on

$$\varphi = -c\, e^{-r} - \frac{1}{6}c^3 e^{-3r}\,, \tag{2.11}$$

$$\mathcal{L} = 1 - (c\, e^{-r})^2 \tag{2.12}$$

exactly. All higher order terms cancel on our solution. No vacuum expectation value is thus turned on. A very similar result was obtained in [18, 19] where it was also found that the definition of the subleading term in the presence of the leading term requires a more careful treatment.

2.5 $\mathrm{SU}(M_1) \times \mathrm{SU}(M_2)$ global symmetry

The third property we would like the spacetime filling D-branes to have, is to come in two distinct sets, giving rise to chiral $\mathrm{SU}(M) \times \mathrm{SU}(M)$ global symmetry, as in QCD. Before we study a chiral example in the next section, let us exhibit this effect in a simpler system, which has a non-chiral $\mathrm{SU}(M_1) \times \mathrm{SU}(M_2)$ global symmetry, the D3-D7-D7' system. The D7 and D7' branes are distinguished by their embedding in the 6 directions transverse to the D3, parametrized by 3 complex variables u_1, u_2, u_3. The D7 branes we studied so far sit at $u_3 = x_8 + i\ x_9 = 0$. D7' branes sit at $u_2 = 0$, breaking another half of the supersymmetries.

The worldvolume gauge theory is $\mathcal{N}=4$ SYM with M_1+M_2 extra hypermultiplets Q and T. Each of the flavors forms a different $\mathcal{N}=2$ subsystem, that is it couples to a different adjoint [20]:

$$W = X[Y,Z] + YQ\tilde{Q} + ZT\tilde{T}\,. \tag{2.13}$$

This way only $\mathcal{N}=1$ supersymmetry is preserved, and the global symmetry is $\mathrm{SU}(M_1)\times\mathrm{SU}(M_2)$. Writing the metric on S^5 as the metric inherited from the C^3 formed out of the 3 $u_i = r_i e^{i\theta_i}$ subject to $\sum r_i^2 = 1$, the two D7 branes are given by the two S^3s u_2=0 and $u_3=0$ respectively. Since they intersect along $u_2=u_3=0$ in C^3, on the S^5 they intersect over a circle. They give rise to two separate spacetime filling 4-branes in AdS$_5$, responsible for the two global symmetries. The reduction of the 8d fields on the equatorial S^3s proceeds as before. On the intersection, the 77' strings give rise to a 6d, bifundamental, hypermultiplet. It's KK reduction on the S^1 gives rise to scalar fields dual to operators mixing Q and T flavors.

From the bulk gravity point of view, a toy model for chiral symmetry breaking is the mass deformation

$$\Delta W = hQ\tilde{T} + \tilde{h}\tilde{Q}T \tag{2.14}$$

This corresponds to giving a vev to the 6d bifundamental hyper [20]. For $M_1=M_2=M$, the global $\mathrm{SU}(M)\times\mathrm{SU}(M)$ symmetry in the field theory gets broken explicitly to the diagonal $\mathrm{SU}(M)$. The 2 disjoint 7 branes join into one smooth 7-brane wrapping the curve [21, 22]

$$u_2u_3 = \epsilon \sim h\tilde{h}\,. \tag{2.15}$$

From the bulk point of view, the gauge symmetry gets higgsed to the diagonal subgroup. The scalars in the 6d hypermultiplet develop a non-trivial profile. This is precisely the way chiral symmetry breaking would be captured by the bulk physics, the only difference being that for chiral symmetry breaking only subleading terms would be present in the profile. A geometric measure for the vev $h\tilde{h}$ is the size of the disk ending on the curve [21]. The curve $u_2u_3=\epsilon$ in R^6 has a non-contractible circle of radius $R=\sqrt{2\epsilon}$. As we will show in the appendix, on the S^5, the corresponding curve still has a non-trivial circle. Its radius at a given value of u is given by $R=\sqrt{2\epsilon}/u$ consistent with having a fermion mass term turned on. To see the absence of a sub-leading term, which would be dual to a fermion-bilinear vev, we would need a better understanding of the non-linear completion of the 6d HM action, that is, the analog of the DBI used previously.

2.6 κ-symmetry

To get a BPS brane in spacetime, supersymmetry variations of the background fields have to be compensated by a κ symmetry transformation on the worldvolume of the D7 branes. For this to work, one needs to satisfy [23]–[27]

$$\Gamma\epsilon = \epsilon \tag{2.16}$$

where

$$\Gamma = e^{-\frac{a}{2}}\Gamma_0' e^{\frac{a}{2}} \tag{2.17}$$

with

$$\Gamma'_0 = i\sigma_2 \otimes \frac{1}{(8)!\sqrt{g}} \epsilon^{i_1 \dots i_8} \partial_{i_1} X^{a_1} \cdots \partial_{i_8} X^{a_8} \, \Gamma'_{a_1 \dots a_8} \,, \tag{2.18}$$

the matrix a captures the B-field and worldvolume gauge field, in our case $B = 0$, $a = 1$. Γ's are curved space Γ-matrices, $\Gamma'_a = E^M_a \Gamma_M$, and ϵ is a Killing spinor of the AdS$_5$ $\times$ S^5 background, whose explicit form can found in [28, 29]. For the metric eq. (2.1) we used in the D3-D7 system:

$$\epsilon = e^{\frac{i}{2}\psi\gamma_5\Gamma^{\psi}} e^{\frac{i}{2}\theta\gamma_5\Gamma^{\theta}} e^{-\frac{i}{2}\alpha_1\Gamma^{\alpha_1\psi}} e^{-\frac{i}{2}\alpha_2\Gamma^{\alpha_2\alpha_1}} e^{-\frac{i}{2}\alpha_3\Gamma^{\alpha_3\alpha_2}} \times R_{AdS}\epsilon_0 \tag{2.19}$$

where $\gamma_5 = \Gamma^{\psi}\Gamma^{\theta}\Gamma^{\vec{\alpha}}$, $\Gamma^{\vec{\alpha}} = \Gamma^{\alpha_1\alpha_2\alpha_3}$, R_{AdS} is a similar rotation matrix to the one we spelled out for the sphere [28, 29] and ϵ_0 is a constant spinor. What is important is that all the sphere Γ matrices as well as γ, the analog of γ_5 for the 5 AdS directions, commute with R_{AdS}.

The projection matrix for the D7 living on the equator, $\psi = \frac{\pi}{2}$, simply becomes

$$\Gamma = -\Gamma^{\vec{\alpha}}\gamma \,. \tag{2.20}$$

Now we want to solve $\Gamma\epsilon = \epsilon$. First, note that since all the exponentials contain an even number of Γ matrices associated with the sphere, γ pulls through all the exponentials and can be made to act on ϵ_0. If we try to do the same with $\Gamma^{\vec{\alpha}}$, we flip the sign of the first exponentials on the left hand side (these are the terms with an odd number of Γ matrices differing from the one we want to pull through). Multiplying both sides with $e^{\frac{-i}{2}\psi\gamma_5\Gamma^{\psi}}$ we arrive at

$$\begin{aligned} -e^{-i\psi\gamma_5\Gamma^{\psi}} e^{\frac{-i}{2}\theta\gamma_5\Gamma^{\theta}} e^{\frac{i}{2}\alpha_1\Gamma^{\alpha_1\psi}} e^{-\frac{i}{2}\alpha_2\Gamma^{\alpha_2\alpha_1}} e^{-\frac{i}{2}\alpha_3\Gamma^{\alpha_3\alpha_2}} \times R_{AdS}\Gamma^{\vec{\alpha}}\gamma\epsilon_0 = \\ = e^{\frac{i}{2}\theta\gamma_5\Gamma^{\theta}} e^{-\frac{i}{2}\alpha_1\Gamma^{\alpha_1\psi}} e^{-\frac{i}{2}\alpha_2\Gamma^{\alpha_2\alpha_1}} e^{-\frac{i}{2}\alpha_3\Gamma^{\alpha_3\alpha_2}} \times R_{AdS}\epsilon_0 \,. \end{aligned} \tag{2.21}$$

At $\psi = \pi/2$, the first term on the lhs just becomes $-i\gamma_5\Gamma^{\psi}$. Pulling this through changes the next two exponentials back to agree with the rhs and we arrive at a simple projector acting on ϵ_0. The D7 brane worldvolume theory hence preserves 16 out of the 32 supercharges of the AdS$_5$ $\times$ S^5 background. In the field theory this reflects the fact that, in the probe limit, the theory with massless flavors still preserves conformal invariance and hence the resulting $\mathcal{N} = 2$ theory actually has 16 instead of 8 supercharges.

Next, let us consider what happens when we turn on the mass parameter, that is we separate the D7 brane from the D3 branes. The superconformal generators now will be broken, but the D7 brane still preserves 8 supercharges. This actually follows from a very general result of Kehagias [30], as discussed in [31]. In a type IIB background of the form

$$ds^2 = Z^{-\frac{1}{2}}\eta_{\mu\nu} + Z^{\frac{1}{2}} ds^2_K \tag{2.22}$$

where the warpfactor Z depends on the internal Ricci flat space K

$$\epsilon = Z^{-\frac{1}{8}}\epsilon_0 \tag{2.23}$$

gives rise to an unbroken supersymmetry for every covariantly constant spinor on K satisfying in addition

$$i\gamma_{0123}\epsilon_0 = \epsilon_0 \,. \tag{2.24}$$

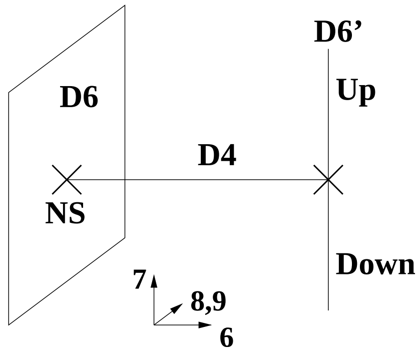

Figure 2: $\mathcal{N}=2$ SYM with $\mathcal{N}=2$ flavors from D6 branes and $\mathcal{N}=1$ flavors from D6' branes.

As shown e.g. in [27], this corresponds precisely to picking out the 16 Poincare supersymmetries of $\mathcal{N}=4$ SYM and breaks the 16 special supersymmetries of the superconformal algebra. In our case, K is flat R^6 and these correspond to the 16 Poincare supersymmetries of AdS$_5$. Writing the background spinor like this, it is obvious that the D7 brane just breaks half of the Poincare supersymmetries for any warpfactor Z, including AdS$_5$ $\times$ S^5, as it does in flat space, since all projection matrices can be pulled through $Z^{\frac{1}{8}}$. The same arguments show, that for the D7-D7' system the 4 supersymmetries that are preserved by the D3-D7-D7' configuration in the flat embedding space are still preserved once we include the backreaction of the D3 branes; that is go over to the warped geometry. In the massless case, in addition we expect to recover 4 more supersymmetries from the special superconformal supersymmetries.

Moreover, it was shown by Grana and Polchinski in [31] that the same supersymmetries are preserved when we turn on, in addition, certain 3-form fluxes on K (the (0,3) piece has to vanish). This includes in particular the Klebanov Strassler solution [32], which we will investigate in the next section.

3. Chiral symmetry and the conifold

3.1 Chiral symmetry from NS5 branes

The problem of how to incorporate chiral flavors in brane setups was first solved in [33] in the context of Hanany-Witten setups [13], which are related to orbifolds of the D3-D7 system by a simple T-duality [34, 35, 36].

Starting with a brane setup for an $\mathcal{N}=2$ gauge theory with NS5 branes along 012345 and D4 branes along 01236, two types of D6 branes can be introduced preserving some supersymmetry. D6 branes along 0123789 add a hypermultiplet worth of matter with the $\mathcal{N}=2$ preserving $QX\tilde{Q}$ superpotential, and hence no chiral symmetry. The rotated D6' brane along 0123457 switches off the superpotential, and hence the gauge theory has the full chiral SU(M) $\times$ SU(M) global symmetry. In the brane setup this is seen as the fact that the D6' brane can split on the NS5 brane, see figure (2). The two pieces by themselves form a HW setup, realizing a 6d SU(M) $\times$ SU(M) gauge theory [37].

3.2 Chiral multiplets from D7 branes on orbifolds

The HW setup we just described has a T-dual description in terms of D3 branes on an C^2/Z_k orbifold. Following a suggestion of [35], [36] showed that for an orbifold acting on u_2 and u_3, the D6 branes map to $u_1 = 0$ while a single D6' brane maps to two D7' branes, one $\mathrm{D7}_3$ along $u_3 = 0$ and one $\mathrm{D7}_2$ along $u_2 = 0$. A standard orbifold calculation [36] yields that each of those D7' branes only gives rise to a single chiral multiplet for a single SU(N) gauge group associated with a single set of fractional branes. If both types of fractional branes are present, like in the cascading solution of KS [32] for which the supergravity background is known, a single D7' gives rise to a chiral fundamental in one and a chiral anti-fundamental in the neighboring gauge group. In more singular geometries obtained from deformation of $Z_{>2}$ orbifolds, the same pattern persists: each D7' brane gives rise to two chiral multiplets, as usual. But they are charged under different gauge groups. If only a single fractional D3 brane is present, the D7' brane only contributes a single chiral multiplet. From the Hanany-Witten point of view this phenomenon was studied in [38] and was called "flavor-doubling". On the blown up orbifold,

$$xy = \prod_{i=1}^{k} (z - z_i) \tag{3.1}$$

the D7' brane wraps the cycle $z = 0$, or $xy = \prod z_i$, a single smooth curve. In the orbifold limit this degenerates into the curve $xy = 0$ with the two branches corresponding to the two different D7' branes. Turning on a mass again corresponds to a vev for the bifundamental scalar of the $\mathrm{D7}_2$ - $\mathrm{D7}_3$, deforming the curve to $xy = \epsilon$, while keeping the $z_i = 0$ (as in our non-chiral toy example).

A new issue that arises in this chiral example is one of anomalies. The number of chiral and anti-chiral flavors has to be the same in the field theory. On the gravity side the new ingredient is that a B-field is turned on in the orbifold background. Even though the 7-branes still wrap a topologically trivial cycle on the base, there now will be induced lower dimensional D-brane charges, and tadpoles become an issue. In the orbifold limit a perturbative string theory calculation [36] shows that tadpoles indeed cancel, if and only if, the number of D7' branes along u_2=0 and $u_3 = 0$ respectively are the same.

3.3 The conifold

In order to get a gravity dual for a confining gauge theory we turn to the conifold. The corresponding gauge theory is pure $\mathcal{N} = 1$ SYM if we study a single set of N fractional 3-branes and the corresponding background was found in [32].[2] In the T-dual brane setup [39, 40], now in addition to rotating the D6 brane we also rotate one of the NS5 branes. So this time, no matter whether we introduce D6 or D6' branes, we will get matter with a chiral symmetry and the corresponding gauge theory is SU(N) SYM with M flavors. The deformed conifold is given by

$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = \text{const.}\,, \tag{3.2}$$

[2]More precisely, the KS solution evolves through a duality cascade through product gauge groups of the form $\mathrm{SU}(N) \times \mathrm{SU}(N + M)$. The physics of the last cascade is (almost) that of pure SYM.

specifying the internal space in the KS solution. The D7 brane, T-dual to the D6' brane which is not affected by the rotation of the NS5 brane, still lives on the curve

$$w_1^2 + w_2^2 = (w_1 + i\, w_2)(w_1 - i\, w_2) = \epsilon\,, \tag{3.3}$$

where ϵ is the mass of the multiplet. For $\epsilon = 0$ we obtain two sets of D7 branes, giving rise to the chiral $\mathrm{SU}(M)\times\mathrm{SU}(M)$ global symmetry, with the bifundamental scalar being dual to the superpotential mass term. As we recalled above, [31] showed that the supersymmetries of the KS background are once more given by the Killing spinors of the internal space multiplied by a power of the warp factor, despite the additional 3-from flux. As shown in [41], κ-symmetry again would require that our 7-brane lie on a holomorphic curve inside the conifold, even in the presence of the B-field. In addition, the pullback of the B-field has to be (1,1) and we would need to verify that it satisfies an extra condition on the worldvolume. Though we have not explicitly done this, we expect from the T-dual setup that the supersymmetric cycle is indeed given by the above curve. In terms of the "usual" coordinates used to describe the singular, warped conifold, i.e. those used in [32], our zero mass cycle is simply given by $\theta_1 = 0$ or $\theta_2 = 0$, very similar to the S^5 case.

4. Conclusions

We have shown that a few flavors can be incorporated into the closed string dual of large-N gauge theories by adding space-time filling probe D-branes. Several unusual properties of D-branes on AdS spaces have been uncovered, most of them due to the dynamics associated with the Breitenlohner-Freedman bound. What remains to be done is a quantitative study of the Klebanov-Strassler solution, which should be possible with the tools we provided. In particular, $\mathcal{N} = 1$ SYM has a runaway superpotential, which should be possible to extract by finding the embedding of the probe D7 branes in the deformed conifold geometry. This runaway may be stabilized by adding a mass term, in which case the scalar dual to the meson vev will have a profile proportional to Λ^3/m. Alternatively, a quartic potential might be part of the field theory dual to the [32] solution with flavors, and a stable vacuum would then exist even for $m = 0$. For this we have to study the supersymmetry conditions for D7 branes in the warped conifold background more closely, we hope to do so in the future. Another theory for which our tools can be applied is pure $\mathcal{N} = 2$ with $\mathcal{N} = 1$ preserving fundamental matter, a theory whose dynamics is rather intricate [42]. Ultimately we believe that our approach will also be useful for non-supersymmetric large-N QCD. Once the supergravity background is known, including a few flavors corresponds to adding two distinct sets of space-time filling D-branes. The chiral condensate corresponds to a nontrivial profile for the bifundamental scalar. The meson-spectrum can be read off from the discrete eigenmodes of the worldvolume gauge fields, just like the glueball masses arise as the eigenmodes of the graviton. The chiral lagrangian appears as the effective action of the zero-modes. Of course, to get $M = N = 3$ the backreaction of the D-branes has to be included. Neglecting the backreaction, we could still compare to quenched lattice calculations.

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A. Image of a holomorphic curve on S^5

Again we want to use eq. (2.15) to determine the curve the single smooth D7 brane wraps in $\mathrm{AdS}_5 \times S^5$ and establish that it is a supersymmetric solution. This time we parametrize the S^5 as

$$\begin{aligned}
\mathrm{Re}(u_1) = x_4 &= u\sin(\psi)\cos(\theta)\\
\mathrm{Im}(u_1) = x_5 &= u\sin(\psi)\sin(\theta)\\
\mathrm{Re}(u_2) = x_6 &= u\cos(\psi)\sin(\xi)\cos(\alpha)\\
\mathrm{Im}(u_2) = x_7 &= u\cos(\psi)\sin(\xi)\sin(\alpha)\\
\mathrm{Re}(u_3) = x_8 &= u\cos(\psi)\cos(\xi)\cos(\beta)\\
\mathrm{Im}(u_3) = x_9 &= u\cos(\psi)\cos(\xi)\sin(\beta)\,.
\end{aligned}$$

The metric is

$$ds^2 = d\psi^2 + \sin^2(\psi)d\theta^2 + \cos^2(\psi)\left(\sin^2(\xi)d\alpha^2 + d\xi^2 + \cos^2(\xi)d\beta^2\right) \tag{A.1}$$

with α, β and θ running from 0 to 2π and ψ and ξ running from 0 to $\frac{\pi}{2}$. The zero mass curve

$$u_2 u_3 = 0 \Leftrightarrow (x_6 = x_7 = 0 \text{ or } x_8 = x_9 = 0) \tag{A.2}$$

is just given by two S^3s

$$\xi = 0 \qquad \text{or} \qquad \xi = \frac{\pi}{2} \tag{A.3}$$

intersecting along the S^1 $\psi = \pi/2$. Turning on ϵ which we choose to be real we get

$$x_6 x_8 - x_7 x_9 = \epsilon \qquad \text{and} \qquad x_6 x_9 + x_7 x_8 = 0 \tag{A.4}$$

which yields

$$u^2\cos(\psi)\sin(2\xi)\sin(\alpha+\beta) = 0 \qquad \text{and} \qquad u^2\cos(\psi)\sin(2\xi)\cos(\alpha+\beta) = 2\epsilon \tag{A.5}$$

and hence, taking α and ξ to be the two fluctuating fields we find the solution

$$\alpha = -\beta \qquad \text{and} \qquad \sin(2\xi) = \frac{2\epsilon}{u^2}\frac{1}{\cos(\psi)}\,. \tag{A.6}$$

As in the D3 D5 case this restricts u to reach a minimal value $u^2_{\mathrm{min}} = 2\epsilon$. In addition for a given $u \geq u_{\mathrm{min}}$ the ψ variable only runs over values such that $\frac{2\epsilon}{u^2\cos(\psi)} \leq 1$. At the boundary, $u \to \infty$, we just recover two equatorial S^3s. From the 5d point of view all that happened is that the bifundamental scalar from the DD' strings develops a profile, higgsing the $\mathrm{SU}(M) \times \mathrm{SU}(M)$ gauge symmetry down to its diagonal subgroup.

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